Econ 802

Final Exam

Greg Dow December 10, 2015

All questions have equal weight. If something is unclear, please ask.

1. Prove that the following statements are true. In each case, assume there are only two inputs. Use graphs whenever this is helpful in explaining your answers.

(a) The Leontief production function has well-defined marginal products at all input bundles <u>except</u> for the cost-minimizing input bundles.

(b) The Cobb-Douglas production function is strictly quasi-concave regardless of whether it has increasing, constant, or decreasing returns to scale.

(c) The CES production function has a constant elasticity of substitution.

2. Answer the following questions mostly using words (a little bit of math is ok).

(a) We often ask whether a profit maximization problem has a solution, but we rarely ask whether a utility maximization problem has a solution. Why?

(b) In the theory of the firm, we distinguish between conditional and unconditional input demand functions. In the theory of the consumer, we distinguish between Hicksian and Marshallian demand functions. Are these concepts related? How?

(c) Define the local elasticity with respect to scale and explain why it is important in the theory of the firm. Then explain why it is <u>not</u> important in consumer theory.

3. Consider the utility function $u(x_1 \ldots x_n)$. Assume this function is strictly quasi-concave and differentiable. Also assume all solutions are interior.

(a) Let e(p, u) be the expenditure function. State and prove Shepherd's Lemma.

(b) Let v(p, m) be the indirect utility function. State and prove Roy's Identity.

(c) Under what conditions, if any, can the Marshallian demand for a good increase when that good's own price increases? Justify your answer.

4. Suppose there are identical firms $j = 1 .. m$. Each has the long run cost function $c(z_j) = z_j^2$ where z_j is firm j's output.

(a) Use a graph to show the long run average and marginal cost curves for a typical firm. Solve for the firm's supply function mathematically and compute the market supply function $S(p)$. If the number of firms is fixed, would you expect each firm to have positive profit? Explain.

(b) Suppose there are identical consumers $i = 1 .. n$. Each has the utility $u(x_i, y_i) = y_i + \ln x_i$ and an endowment $w > 0$ of the y good. There are no endowments of the x good. The price of y is $p_y \equiv 1$ and the price of x is p. Let z_j be firm j's output of x, with cost $c(z_j)$ measured in units of y. Derive the market demand $D(p)$. Then use $S(p)$ from part (a) to solve for the equilibrium price p^* and quantity $X^* = D(p^*) = S(p^*)$. Is this equilibrium Pareto efficient? Carefully justify your answer.

(c) Now the government imposes a tax $t > 0$ per unit of the x good, which is paid by the firms. Using a graph, show the resulting consumer surplus, producer surplus, tax revenue, and deadweight loss. Write down an equation that could be used to solve for the supply price and demand price as functions of the tax rate and other exogenous parameters. You don't need to solve explicitly for these prices but be sure to explain what the steps in the calculation would be.

5. Consider a pure exchange model with two consumers (A, B) and two goods (1, 2). A has the utility $u_A = \ln x_{A1} + \ln x_{A2}$ and B has the utility $u_B = cx_{B1} + dx_{B2}$ where $c > 0$ and $d > 0$. The aggregate endowments of the goods are $w_1 > 0$ and $w_2 > 0$.

(a) Draw an Edgeworth box showing typical indifference curves for A and B. Then derive an equation that describes the contract curve, and show the contract curve in your graph. Does the contract curve necessarily pass through the origin points for both A and B? Explain briefly.

(b) Suppose A has the endowment vector $(w_1, 0)$ and B has the endowment vector $(0, w_2)$. Find a Walrasian equilibrium price vector (p_1, p_2) and calculate equilibrium consumption bundles for A and B. Is zero degree homogeneity of the aggregate excess demand function relevant in this model? Is Walras's Law relevant in this model? Explain briefly in each case.

(c) Now suppose there are n_A consumers of type A and n_B consumers of type B. The aggregate endowments are $W_1 > 0$ and $W_2 > 0$. A benevolent social planner wants to maximize total utility subject to feasibility constraints on the goods. Everyone of type A receives the same bundle (x_{A1}, x_{A2}) and everyone of type B receives the same bundle (x_{B1}, x_{B2}). Solve for the planner's optimal allocation. Then show that for appropriate choices of the prices and the individual endowments, the planner's desired allocation is a Walrasian equilibrium.